STUDY GUIDE:

Module 5: Rational Numbers, Part 2

In this module we continue the work we began in Module 4. Recall that in Module 4 we restricted ourselves to cases in which we took fractionl parts of numbers that were multiples of the denominator. For example, we looked at $\frac{2}{3}$ of 6 or $\frac{2}{3}$ of 9; but we didn't look at $\frac{2}{3}$ of 7 or $\frac{2}{3}$ of 8. In this module we consider such ratios as $\frac{2}{3}$ of 7. This requires that we be able to divide 7 by 3. To do this we think in terms of lengths rather than in terms of tally marks. That is, we can divide a 7 inch length into three parts of equal length, but we can't divide 7 tally marks into three equal parts without having to deal with remainders.

Once we've generalized the idea of fractional parts, we turn to the question of multiplying and dividing common fractions. We show that, for example, $\frac{3}{4}$ of $\frac{5}{7}$ is $\frac{15}{28}$; and once we see that the problem required that we multiply numerators and multiply denominators, we define $\frac{3}{4}$ of $\frac{5}{7}$ to mean $\frac{3}{4} \times \frac{5}{7}$ or $\frac{3 \times 5}{4 \times 7}$.

Finally, we show that division is still the "inverse" of multiplication in the sense that $\frac{2}{3} \div \frac{5}{7} = \_\_\_\_\_$ means $\frac{2}{3} = \frac{5}{7} \times \_\_\_\_\_$ and we see how to divide two common fractions in terms of knowing how to multiply two common fractions.

Using the results of Modules 4 and 5 combined, by the end of this module we are able to add, subtract, multiply, and divide rational numbers using the language of common fractions.

Step 1:

View Videotape Lecture #5.

Step 2:

Read Module 5 of the text.

Step 3:

When you feel you understand the material presented in Steps 1 and 2, complete the following "Check-The-Main-Ideas" self-quiz by correctly filling in each blank.

Check the Main Ideas:

In this module we explain common fractions

in terms of the arithmetic operation of ________. *division*

That is, for example, $\frac{2}{3}$ means 2 _______ by 3. In *divided*

terms of lengths, we may view $\frac{2}{3}$ as the length of

each piece if a _____ inch length is divided into *2*

_____ pieces of equal length. *3*

If we think of 2 as an adjective, say as in

2 dozen doughnuts; we may replace 2 dozen by the

number ____. Hence, if we divide 2 dozen equally *24*

among 3 people we get:

2 dozen doughnuts ÷ 3 people =

24 doughnuts ÷ 3 people =

(24 ÷ 3) ______ per _______ = *doughnuts; person*

_____ doughnuts per person. *8*

Since it takes 12 doughnuts to make a dozen,

8 doughnuts is ___ twelfths of a dozen. In lowest *8*

terms, $\frac{8}{12}$ is _____. Hence 2 dozen doughnuts divided *$\frac{2}{3}$*

by 3 people is _____ of a dozen per person. *$\frac{2}{3}$*

The word in "doughnuts per person" that tells us we have a rate problem is ______. More generally in terms of arithmetic, whenever the word "per" appears between two nouns we may replace it by the arithmetic symbol, _____.

"per"

$\div$

For example since $200 \div 4 = 50$, if a car travels 200 miles in 4 hours we say that its average speed is 50 ___________.

miles per hour

Sometimes we take a fractional part of a fractional part. Suppose we want to take $\frac{4}{9}$ of $\frac{5}{7}$. The least common multiple of 9 and 7 is ______. So let's assume that we're dealing with 63 parts. $\frac{5}{7}$ of 63 is $(63 \div$ ___$) \times 5$ or 45. Hence

63

7

$$\frac{4}{9} \text{ of } \frac{5}{7} \text{ of } 63 =$$

$$\frac{4}{9} \text{ of } \_\_\_\_\_$$

45

Since $45 \div 9 = 5$ and $5 \times 4 = 20$, $\frac{4}{9}$ of $45 =$ ______.

20

In other words, $\frac{4}{9}$ of $\frac{5}{7}$ of $63 = 20$. Using common fractions, we see that:

$$\frac{4}{9} \text{ of } \frac{5}{7} = \_\_\_\_\_\_$$

$\frac{20}{63}$

Now we replace "of" by "X" and we see that

$$\frac{4}{9} \times \frac{5}{7} = \_\_\_\_\_\_$$

$\frac{20}{63}$

We could get the same answer more mechanically by taking the product of 4 and _____ to get the numerator , and the product of 9 and ___ to get the _________.

5

7

denominator

By way of another example, since $3 \times 6 = 18$

and $5 \times 7 = 35$, $\frac{3}{5} \times \frac{6}{7} =$ ____. That is, $\frac{18}{35}$

$\frac{3}{5}$ of $\frac{6}{7} =$ _____. Since $3 \times 6 = 6 \times 3$ and $5 \times 7 = 7 \times 5$, $\frac{18}{35}$

$\frac{6}{7}$ of $\frac{3}{5}$ is also _______. $\frac{18}{35}$

Division is another form of multiplication.

For example $\frac{3}{5} \div \frac{2}{7}$ means the number we must

multiply by _____ to get ______ as the quotient. *2/7; 3/5*

A quick way of finding the answer is to multiply

$\frac{3}{5}$ by ______. That is: *7/2*

$$\frac{3}{5} \div \frac{2}{7} =$$

$$\frac{3}{5} \times \_\_ = \qquad \frac{7}{2}$$

$$\frac{21}{10}$$

Now suppose we wanted to find the number

that had to be multiplied by $\frac{4}{9}$ to give $\frac{11}{13}$ as

the product. In terms of division this would

be written as _____ ÷ _____. Then *11/13; 4/9*

to find the quotient we would multiply $\frac{11}{13}$ by

_______. In other words, we would multiply *9/4*

$\frac{4}{9}$ by ___ to get 1 and then we'd multiply 1 by *9/4*

____ to get $\frac{11}{13}$. *11/13*

But we must be careful not to confuse

$\frac{11}{13} \div \frac{4}{9}$ with $\frac{4}{9} \div \frac{11}{13}$. $\frac{4}{9} \div \frac{11}{13}$ means the

number we must multiply by _____ to get *11/13*

_______ as the product. *4/9*

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Step 4:

Do the Mastery Review on the next page.

Mastery Review

Answers:

1. The cost of an $11 gift is shared equally by 4 people. How much money did each person pay? — 1. ______

2. The cost of an $11 gift is to be shared by 6 people. Is it possible for all 6 to pay exactly the same amount? — 2. ______

3. 6 people decide to share an 11 hour shift so that all 6 work exactly the same length of time. How long does each person work? — 3. ______

4. If 5 grapefruit cost $2, what is the average cost of each grapefruit? — 4. ______

5. What rational number is named by $\frac{20}{4}$? — 5. ______

6. (a) The cost of a $3 item is shared equally by 4 people. How much does each person pay? — 6. (a) ______

   (b) How much is $\frac{3}{4}$ of $1? — (b) ______

7. How much is $\frac{4}{9}$ of $\frac{2}{5}$ of 180? — 7. ______

7(a). Write the product of $\frac{4}{9}$ and $\frac{2}{5}$ as a common fraction. — 7(a). ______

8. Your friend buys a coat for 2/5 of the regular price and sells it to you for 4/9 **of** the price he paid. What portion of the regular price did you pay? — 8. ______

9. Write $\frac{5}{6} \times \frac{18}{25}$ as a common fraction in lowest terms. — 9. ______

10. How much is $\frac{4}{7}$ of 1? — 10. ______

11. Which names the greater ratio, $\frac{4}{7} \times \frac{5}{9}$ or $\frac{5}{9} \times \frac{4}{7}$? — 11. ______

12. How much is $\frac{8}{5} \times \frac{2}{3}$? — 12. ______

13. How much is $\frac{7}{7} \times \frac{3}{4}$? — 13. ______

14. What whole number is named by $\frac{7}{1}$? — 14. ______

15. How much is $\frac{2}{5}$ of 19? — 15. ______

(cont)

Mastery Review (cont)

Answers:

16. What is the product of $\frac{4}{7}$ and $\frac{7}{4}$? 16. ______

17. What is the multiplicative inverse of 9? 17. ______

18. Write each of the following as a common fraction in lowest terms: 18. (a) ______ (b) ______

    (a) $\frac{2}{9} \times (\frac{3}{5} \times \frac{4}{7})$

    (b) $(\frac{2}{9} \times \frac{3}{5}) \times \frac{4}{7}$

19. What common fraction is named by $(\frac{5}{6} \times \frac{3}{4}) \times \frac{4}{3}$? 19. ______

20. Use the result of Exercise 19 to fill in the blank: 20. ______

    $\_\_\_\_ \times \frac{4}{3} = \frac{5}{6}$

21. What is the quotient when $\frac{4}{5}$ is divided by $\frac{3}{7}$? 21. ______

22. What is the quotient when $\frac{3}{7}$ is divided by $\frac{4}{5}$? 22. ______

23. How much is $\frac{4}{7} \div 5$? 23. ______

24. How much is $15 \div \frac{1}{5}$? 24. ______

25. How much is $\frac{1}{5} \div \frac{1}{25}$? 25. ______

26. Find the value of 4/7 ÷ 3/11 by converting both common fractions to common denominators. 26. ______

27. You can buy 3/5 pounds of apples for 27¢. At this rate how much would a pound of apples cost? 27. ______

Answers:

1. $2.75   2. No   3. 110 minutes (or 1 hour and 50 minutes)   4. 40¢

5. 5   6. (a) 75¢ (b) 75¢   7. 32   7(a). 8/45   8. 8/45

9. 3/5   10. 4/7   11. They're equal   12. 16/15   13. 3/4

14. 7   15. 38/5   16. 1   17. 1/9   18. (a) 8/105 (b) 8/105

19. 5/6   20. 15/24 or 5/8   21. 28/15   22. 15/28   23. 4/35   24. 75

25. 5   26. 44/21   27. 45¢

Step 5:

Do Self-Test 5, Form A

Self-Test 5, Form A

1. Which of the following quotients names the greatest rational number:

   (a) $2 \div 3$ (b) $3 \div 4$ (c) $5 \div 12$?

2. Which of the following represents the cheapest price per pound: (a) 50 pounds for \$29.50 (b) 60 pounds for \$34.20 or (c) 40 pounds for \$24.40?

3. (a) How much is $\frac{4}{11} \times \frac{3}{7}$?

   (b) What must you multiply $\frac{4}{11}$ by to get $\frac{3}{7}$?

4. Which is more and by how much, 3/4 of 2/7 or 5/6 of 3/14?

5. A recipe calls for 2/3 of a cup of flour. You only want to make 3/4 of the recipe. How much flour should you use?

6. Your friend buys 1/7 of a carton of books. You buy 2/5 of what's left. What fractional part of the carton did you buy?

7. Write each of the following as a common fraction in lowest terms:

   (a) $\frac{2}{3} \times (\frac{4}{5} + \frac{1}{7})$ (b) $(\frac{2}{3} \times \frac{4}{5}) + (\frac{2}{3} \times \frac{1}{7})$

8. Write each of the following as a common fraction in lowest terms:

   (a) $(\frac{2}{3} \div \frac{6}{7}) \div \frac{3}{7}$ (b) $\frac{2}{3} \div (\frac{6}{7} \div \frac{3}{7})$

9. An object travels 2/5 of a mile in 3/7 of a minute. What is the speed of the object in:

   (a) miles per minute? (b) miles per hour?

10. A map uses a scale of 1/4 of an inch to represent 75 feet. How many feet is represented by:

    (a) 1 inch? (b) 2/3 of an inch?

ANSWERS:

1. ________
2. ________
3. (a) ________ (b) ________
4. ________
5. ________
6. ________
7. (a) ________ (b) ________
8. (a) ________ (b) ________
9. (a) ________ (b) ________
10. (a) ________ (b) ________

(ANSWERS ARE ON THE NEXT PAGE)

Answers for Self-Test 5, Form A

1. (b)

2. (b)

3. (a) $\frac{12}{77}$ (b) $\frac{33}{28}$

4. $\frac{2}{7} \times \frac{3}{4}$ by $\frac{1}{28}$

5. 1/2 of a cup

6. $\frac{12}{35}$

7. (a) $\frac{22}{35}$ (b) $\frac{22}{35}$

8. (a) $\frac{49}{27}$ (b) $\frac{1}{3}$

9. (a) $\frac{14}{15}$ miles per minute (b) 56 miles per hour

10. (a) 300 feet (b) 200 feet.

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If you did each problem in Form A correctly, you may, if you wish, proceed to the next module. Otherwise continue with Step 6.

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Step 6:

Study the solutions to Self-Test 5, Form A with special emphasis on any problems you failed to answer correctly.

Solutions for Self-Test 5, Form A

1.

Actually, this is a rephrasing of an exercise similar to one we did in Self-Test 4. The point is that $2 \div 3$ means $\frac{2}{3}$; $3 \div 4$ means $\frac{3}{4}$; and $5 \div 12$ means $\frac{5}{12}$. So converting to common fractions and using common denominators we have:

Here we're trying to show that $2 \div 3$, for example, has the same meaning as at a rate of 2 out of each 3.

$$2 \div 3 = \frac{2}{3} = \frac{8}{12}$$

$$3 \div 4 = \frac{3}{4} = \frac{9}{12}$$

$$5 \div 12 = \frac{5}{12}$$

If we didn't want to use common fractions, we could think in terms of:

2 dozen ÷ 3 people = 24 ÷ 3 people = 8 per person

3 dozen ÷ 4 people = 36 ÷ 4 people = 9 per person

5 dozen ÷ 12 people = 60 ÷ 12 people = 5 per person

So 3 ÷ 4 is the greatest (9 per 12) followed by 2 ÷ 3 (8 per 12) and 5 ÷ 12 (5 per 12) is the least.

Here we're using the fact that numbers, whether whole numbers or fractions, can be viewed as adjectives modifying nouns. With the right choice of nouns, the fractions can be translated into whole numbers.

2.

Remember that "per" is the key word. In this case, "price *per* pound" means "price ÷ pounds" To avoid the use of the decimal point in such numbers as $29.50, we may convert dollars to cents by moving the decimal point two places to the right to get 2,950 cents. Now if we divide the number of cents by the number of pounds, the denomination will be <u>cents *per* pound</u> (cents *divided by* pounds).

Decimal fractions will be discussed in Modules 7 and 8. But for now notice that because there are 100 cents per dollar, $29 is 100 cents 29 times or 2,900 cents. Adding 50 cents gives us 2,950 cents.

2. (cont)

Now we're ready to do Exercise 2. Let's compute the number of cents per pound in all three parts. We get:

(a) 50 pounds for \$29.50 =

50 pounds *per* \$29.50 =

\$29.50 per 50 pounds =

2,950 cents per 50 pounds =

2,950 cents ÷ 50 pounds =

2,950 ÷ 50 *cents per pound* =

59 cents per pound

We have to paraphrase and replace "for" by "per" in order to prepare for division

Order is important. We want cents per pound <u>not</u> pounds per cent.

```
      59
50)2,950
   2 50
     450
   - 450
```

(b) 60 pounds for \$34.20 =

60 pounds per \$34.20 =

\$34.20 per 60 pounds =

3,420 cents per 60 pounds =

3,420 cents ÷ 60 pounds =

3,420 ÷ 60 cents per pound =

57 cents per pound

You don't have to supply all these steps. They are included only as a guide in case you have trouble.

```
      57
60)3,420
  -3 00
     420
   - 420
```

(c) 40 pounds for \$24.40 =

40 pounds per \$24.40 =

\$24.40 per 40 pounds =

2,440 cents per 40 pounds =

2,440 cents ÷ 40 pounds =

2,440 : 40 cents per pound =

61 cents per pound

We can't tell the best price simply by looking at the total price.

The comparison requires that we compare prices on a common amount.

```
      61
40)2,440
  - 2 40
      40
    - 40
```

So in terms of "unit pricing" - in this case, cents per pound- we see that (b) is the cheapest rate.

3.

This problem compares the relationship between multiplication and division. Notice that in both parts (a) and (b) reference is made to multiplication, but that (b) is really a division problem.

(a)

To multiply two fractions we multiply the numerators to get the numerator of the product and we multiply the two denominators to get the denominator of the product. Hence:

$$\frac{4}{11} \times \frac{3}{7} = \frac{4 \times 3}{11 \times 7} = \frac{12}{77}$$

4/11 of 3/7 of 77 =
4/11 of (3/7 of 77) =
4/11 of 33 =
12

(b)

In terms of fill-in-the-blank, the problem has the form:

$$\frac{4}{11} \times \_\_\_\_ = \frac{3}{7} \qquad (1)$$

(1) means the same as

$$\frac{3}{7} \div \frac{4}{11} = \_\_\_\_ \qquad (2)$$

If the fractions confuse you, think in terms of whole numbers. For example 2 X __ = 6 suggests 6 ÷ 2 not 2 ÷ 6. That is: $f \times \_\_\_ = s$

means

$\_\_\_ = s \div f$

Using the "invert-and-multiply" rule for division we have:

$$\frac{3}{7} \underset{\downarrow}{\div} \underset{\downarrow}{\frac{4}{11}} =$$

$$\frac{3}{7} \times \frac{11}{4} =$$

$$\frac{3 \times 11}{7 \times 4} =$$

$$\frac{33}{28}$$

Once the problem has been rewritten as a multiplication problem, we use the rules for multiplication.

Solutions for Self-Test 5, Form A (cont)

3 (b). (cont)

If you tend to confuse $\frac{3}{7} \div \frac{4}{11}$ with $\frac{4}{11} \div \frac{3}{7}$ try to proceed logically from (1). Namely, to see what we must multiply $\frac{4}{11}$ by to get $\frac{3}{7}$, first multiply $\frac{4}{11}$ by $\frac{11}{4}$ to get 1; and then multiply 1 by $\frac{3}{7}$ to get $\frac{3}{7}$. That is:

$$\underbrace{\left(\frac{4}{11} \times \frac{11}{4}\right)}_{1} \times \frac{3}{7} = \frac{3}{7}$$

and by the asociative property we can rewrite

$$\left(\frac{4}{11} \times \frac{11}{4}\right) \times \frac{3}{7} \text{ as } \frac{4}{11} \times \underbrace{\left(\frac{11}{4} \times \frac{3}{7}\right)}_{\frac{33}{28}}$$

No matter what method you use, you can always check your answer. Namely if we did the problem correctly 4/11 X 33/28 should equal 3/7. Well:

$$\frac{4}{11} \times \frac{33}{28} = \frac{4 \times 33}{11 \times 28} =$$

$$= \frac{4 \times 3 \times 11}{11 \times 4 \times 7}$$

$$= \frac{\cancel{4} \times 3 \times \cancel{11}}{\cancel{11} \times \cancel{4} \times 7}$$

$$= \frac{3}{7}$$

No other answer will check.

A major point of this exercise is to help you see how important it is to read a problem correctly. While parts (a) and (b) may look similar, our answers show us that the two parts are very different.

4.

This exercise involves some principles from Module 4 and the recognition that "of" means "X". Namely:

$$\frac{3}{4} \text{ of } \frac{2}{7} = \frac{3}{4} \times \frac{2}{7}$$

$$= \frac{3 \times 2}{4 \times 7}$$

$$= \frac{3 \times \cancel{2}}{\cancel{2} \times 2 \times 7}$$

$$= \frac{3}{14}$$

Our strategy will be to write each expression as a common fraction, convert to a common denominator, and then compare numerators.

4. (cont)

Note that we did not have to cancel first. Had we wished, we could have written:

$$\frac{3 \times 2}{4 \times 7} = \frac{6}{28} \qquad (1)$$

and then cancelled 2 from both numerator and denominator to get $\frac{3}{14}$.

In fact, as we shall soon see, there is no particular value in this exericise to reduce our answer to lowest terms. What will be important is that we have a common denominator.

$$\frac{5}{6} \text{ of } \frac{3}{14} = \frac{5}{6} \times \frac{3}{14}$$

$$= \frac{5 \times 3}{6 \times 14}$$

$$= \frac{5 \times 3}{(2 \times 3) \times (2 \times 7)}$$

$$= \frac{5 \times \cancel{3}}{2 \times \cancel{3} \times 2 \times 7}$$

$$= \frac{5}{28} \qquad (2)$$

Since 3/4 of 2/7 is 3/14 and since 6/7 of 3/14 is less than 3/14 we already know that 5/6 of 3/14 is less than 3/4 of 2/7. What we're working on now is the actual difference.

Comparing (1) and (2) we see that $\frac{3}{4}$ of $\frac{2}{7}$ exceeds $\frac{5}{6}$ of $\frac{3}{14}$ by $\frac{1}{28}$ *(that is, 6/28 - 5/28 = 1/28)*

Granted that the whole numbers in $\frac{5}{6}$ of $\frac{3}{14}$ seem greater than those in $\frac{3}{4}$ of $\frac{2}{7}$, the key point in determining the size of a ratio is the size of the numerator compared with the size of the denominator.

3/4 of 2/7(of 28) =
3/4 of (2/7 of 28) =
3/4 of 8 =
6 (of 28)

5/6 of 3/14 (of 28) =
5/6 of (3/14 of 28) =
5/6 of 6 =
5 (of 28)

So 3/4 of 2/7 exceeds 5/6 of 3/144 by 1 part in 28

5.

This is simply an application of multiplying fractions. Namely what we want here is to find out how much $\frac{3}{4}$ of $\frac{2}{3}$ of a cup of flour is. So all we have to do is compute $\frac{3}{4}$ of $\frac{2}{3}$. We get:

To take 3/4 of a number means that we divide the number by 4 and then multipl the quotient by 3. But the overall process is multiplication, not division!

$$\frac{3}{4} \text{ of } \frac{2}{3} = \frac{3}{4} \times \frac{2}{3}$$

$$= \frac{3 \times 2}{4 \times 3}$$

5. (cont)

$$= \frac{\cancel{3} \times \cancel{2}}{\cancel{2} \times 2 \times \cancel{3}}$$

$$= \frac{1}{2}$$

What we're doing is first taking 1/4 of 2/3 (giving us 2/12 or 1/6) and we're then taking 1/6 three times to get 3/6 or 1/2

Therefore $\frac{3}{4}$ of $\frac{2}{3}$ of a cup of flour is the same as $\frac{1}{2}$ of a cup of flour.

Many people who can't handle fractions, simply make the whole recipe and then serve bigger portions or else serve the rest as leftovers.

6.

Since your friend bought $\frac{1}{7}$ of the carton, $\frac{6}{7}$ of the carton still remain. Since you're buying $\frac{2}{5}$ of what's left, you're buying

$\frac{2}{5}$ of $\frac{6}{7}$ of a carton

To compute $\frac{2}{5}$ of $\frac{6}{7}$, we have:

$$\frac{2}{5} \text{ of } \frac{6}{7} = \frac{2}{5} \times \frac{6}{7}$$

$$= \frac{2 \times 6}{5 \times 7}$$

$$= \frac{12}{35}$$

This is another application, but this time we have to do 2 steps with fractions. We must first subtract the 1/7 of the carton your friend bought and then we take 2/5 of the remainder.

To translate this problem into whole numbers, imagine that there are 35 books in a carton. Then 1/7 of a carton is 1/7 of 35 books or 5 books. If your friend buys 5 books there are still 30 left in the carton. Hence you are buying 2/5 of 30 books or 2 X (30 ÷ 5) or 12 books. That is you'd be buying 12 of the 35 books.

In terms of whole numbers, you are purchasing 12 of every 35 books that are in the carton.

Notice that since we can't have a fractional part of a book, this exercise requires that the number of books in each carton be a multiple of 35.

7.

The aim of this exercise, aside from providing drill in the arithmetic of common fractions, is to show that the distributive property is obeyed even when we deal with common fractions.

Recall that the distributive property has the form:

$$f \times (s + t) = (f \times s) + (f \times t)$$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ \frac{2}{3} & \frac{4}{5} & \frac{1}{7} \end{array}$$

7. (cont)

Remember to do the arithmetic within the parentheses first.

(a)

$$\frac{4}{5} = \frac{4 \text{ X } 7}{5 \text{ X } 7} = \frac{28}{35}$$

$$\frac{1}{7} = \frac{1 \text{ X } 5}{7 \text{ X } 5} = \frac{5}{35}$$

We're preparing to add 4/5 and 1/7 by the method discussed in Module 4.

Therefore:

$$\frac{2}{3} \text{ X } (\frac{4}{5} + \frac{1}{7}) = \frac{2}{3} \text{ X } (\frac{28}{35} + \frac{5}{35})$$

$$= \frac{2}{3} \text{ X } (\frac{28 + 5}{35})$$

$$= \frac{2}{3} \text{ X } \frac{33}{35}$$

$$= \frac{2 \text{ X } 33}{3 \text{ X } 35}$$

$$= \frac{2 \text{ X } \overset{1}{\cancel{3}} \text{ X } 11}{\underset{1}{\cancel{3}} \text{ X } 5 \text{ X } 7}$$

$$= \frac{22}{35}$$

(b)

$$\frac{2}{3} \text{ X } \frac{4}{5} = \frac{2 \text{ X } 4}{3 \text{ X } 5} = \frac{8}{15} = \frac{8 \text{ X } 7}{15 \text{ X } 7} = \frac{56}{105}$$

$$\frac{2}{3} \text{ X } \frac{1}{7} = \frac{2 \text{ X } 1}{3 \text{ X } 7} = \frac{2}{21} = \frac{2 \text{ X } 5}{21 \text{ X } 5} = \frac{10}{105}$$

15 = 3 X 5 and 21 = 3 X 7. Hence the least common multiple of 15 and 21 is 3 X 5 X 7 or 105. We could use 21 X 15 or 315 as a common denominator but 105 makes the arithmetic easier.

Hence:

$$(\frac{2}{3} \text{ X } \frac{4}{5}) + (\frac{2}{3} \text{ X } \frac{1}{7}) =$$

$$\frac{56}{105} + \frac{10}{105} =$$

$$\frac{66}{105} =$$

6 + 6 = 12 and 1 + 0 + 5 = 6. Since both 12 and 6 are divisible by 3, so also are 66 and 105.

$$\frac{3 \text{ X } 22}{3 \text{ X } 35} =$$

$$\frac{22}{35}$$

8.

The aim of this exercise, aside from giving you experience with division of fractions, is to show you the importance of grouping symbols when we divide fractions. In essence, the fact that parts (a) and (b) have different answers shows that division of fractions does not have the associative property.

That is, if you remove the parentheses in parts (a) and (b), the two problems look identical.

(a)

$$\frac{2}{3} \div \frac{6}{7} =$$

$$\frac{2}{3} \times \frac{7}{6} =$$

$$\frac{2 \times 7}{3 \times 6} =$$

$$\frac{14}{18} =$$

$$\frac{7}{9}$$

$$\frac{14}{18} = \frac{2 \times 7}{2 \times 9} = \frac{7}{9}$$

Therefore:

$$\left(\frac{2}{3} \div \frac{6}{7}\right) \div \frac{3}{7} = \frac{7}{9} \div \frac{3}{7}$$

$$= \frac{7}{9} \times \frac{7}{3}$$

$$= \frac{49}{27}$$

Remember to work inside the parentheses first.

(b) $\frac{2}{3} \div \left(\frac{6}{7} \div \frac{3}{7}\right) =$

$$\frac{2}{3} \div \left(\frac{6}{7} \times \frac{7}{3}\right) =$$

$$\frac{6}{7} \times \frac{7}{3} = \frac{6 \times 7}{7 \times 3} = \frac{2 \times \cancel{3} \times \cancel{7}}{\cancel{7} \times \cancel{3}}$$

$$\frac{2}{3} \div 2 =$$

$$\frac{2}{3} \times \frac{1}{2} =$$

$$\frac{1}{3}$$

Remember that 2 is the same as 2/1 and that the reciprocal of 2/1 is 1/2.

9.

The key here is to study the denominations. "Miles per minute" suggests "miles ÷ minutes, while "miles per hour" suggests "miles ÷ hours". Hence:

The word "per" coming between two nouns tells us to divide.

(a)

$$\frac{2}{5} \text{ miles} \div \frac{3}{7} \text{ minutes} =$$

$$\frac{2}{5} \div \frac{3}{7} \text{ miles per minute} =$$

$$\frac{2}{5} \times \frac{7}{3} \text{ miles per minute} =$$

$$\frac{14}{15} \text{ miles per minute.}$$

Remember that 2/5 miles means the same thing as 2/5 of a mile and that 3/7 minutes means the same as 3/7 of a minute.

If you don't like fractions, it might help to notice that 14/15 miles per minute says the same thing as 14 miles per 15 minutes. When we say that we went 14 miles every 15 minutes it doesn't seem like we're talking about fractions.

(b) The missing piece of information here is that there are 60 minutes per hour. So since the object travels $\frac{14}{15}$ miles each minute, in 60 minutes (1 hour) it travels $\frac{14}{15}$ miles, 60 times, or:

$$\frac{14}{15} \text{ miles} \times 60 =$$

$$\frac{14}{15} \times \frac{60}{1} \text{ miles} =$$

$$\frac{14}{\underset{1}{\cancel{15}}} \times \frac{\overset{4}{\cancel{60}}}{1} = \frac{56}{1} \text{ miles.}$$

See how helpful it is to think in terms of 14 miles each 15 minutes? Namely: 14 miles per 15 minutes = 28 miles per 30 minutes = 42 miles per 45 minutes = 56 miles per 60 minutes, and so on. The miles are multiples of 14 and the minutes are multiples of 15.

In other words, $\frac{14}{15}$ miles per minute is the same rate as 56 miles per hour.

Be careful to notice that we aren't saying that the object went 56 miles in 1 hour. All we're saying is that at a rate of 2/5 miles per 3/7 minutes, the object would have gone 56 miles in one hour.

Make sure you label the answer. Do not write 56 or 14/15. An adjective must modify a noun. We have to tell 56 "what"; in this case, 56 miles per hour.

Of course, if someone says how many miles an hour did you travel, then "56" is an acceptable reply because the question states "miles per hour"

9. (cont)

As a concluding remark to this exercise, we should comment about *relative* size. For example, $\frac{2}{5}$ of a mile may not seem like a very great distance. By the same token, $\frac{3}{7}$ of a minute is not a very long time. *As we've indicated in the margin note, an hour is more than* 120 times *greater than 3/7 minutes.* So if the object goes 2/5 miles in one 3/7 of a minute interval, it will go more than 120 X $\frac{2}{5}$ miles in an hour. 120 ÷ 5 = 24 and 24 X 2 = 48. Hence the object will travel more than 48 miles in an hour.

The point is that when we see such small numbers as 2/5 miles and 3/7 minutes, it is usually hard for us to visualize such a large answer as 56 miles per hour.

Since 3/7 X 2 = 6/7 which is less than 1, 3/7 of a minute occurs more than twice in a minute. Hence in an hour it occurs more than 2 X 60 or 120 times. To find the exact number of times, we should divide 60 by 3/7. Then we should multiply this quotient by 2/5 to find out how far the object would have moved in an hour:

$$60 \div 3/7 = 60 \times 7/3 = 20 \times 7 = 140$$

Hence there are exactly 140 3/7 of a minute time intervals per hour. Since each time the object moves 2/5 of a mile, in one hour the object will move 2/5 X 140 or 2 X 28 or 56 miles.

Dividing 2/5 miles by 3/7 minutes was just a quicker way of getting the same result.

In summary we aren't interested in how far the object moved nor in how long it moved. What we are interested in is what was the rate of change of distance with respect to the change in time (This is the definition of speed); and we've shown that if the object were to keep moving at the given rate it would have gone 56 miles in one hour.

10.

Here is another application in which it is important to understand how to multiply and divide fractions. The problem tells us that each 1/4 of an inch represents 75 feet. So counting by multiples of 1/4 we can find out how many feet are represented by any number

When we write 1/4, 2/4, 3/4, and so on, it looks like fractions; but if we write 1 fourth, 2 fourths, 3 fourths, and so on, it looks like whole numbers.

10. (cont)

of multiples of $\frac{1}{4}$ inches. That is:

1 fourth of an inch represents 75 feet.
2 fourths of an inch represents 150 feet.
3 fourths of an inch represents 225 feet.
4 fourths of an inch represents 300 feet.
5 fourths of an inch represents 375 feet.
and so on

(a) Since an inch is 4 fourths of an inch, we see from the above chart that 1 inch represents 300 feet.

4 fourths means 4/4 and 4/4 = 1.

If we hadn't made the chart, we could have taken 1 inch and divided it by 1/4 of an inch to get:

$$1 \div \frac{1}{4} = 1 \text{ X } \frac{4}{1} = 1 \text{ X } 4 = 4$$

This tells us that there are four 1/4 of an inch in 1 inch. We then multiply 75 feet by 4 and get 300 feet as our answer.

This would be a long way to do part (a) but in part (b) we have 2/3 of an inch which is not a multiple of 1/4 of an inch. In that case, it is important to understand division.

(b)

If we want to use the result of part (a), we can say that since 1 inch represents 300 feet, $\frac{2}{3}$ of an inch will represent $\frac{2}{3}$ of 300 feet; and

$$\begin{aligned}\frac{2}{3} \text{ of } 300 \text{ feet} &= 2 \text{ X } (300 \text{ feet} \div 3)\\ &= 2 \text{ X } 100 \text{ feet}\\ &= 200 \text{ feet.}\end{aligned}$$

But suppose we hadn't have already done part (a) and we wanted to solve part (b) using only the fact that 1/4 of an inch represents 75 feet and that we had a length of 2/3 inches.

Solutions for Self-Test 5, Form A (concluded)

10 (b). (cont)

The idea is to divide $\frac{2}{3}$ by $\frac{1}{4}$. This will tell us how many times $\frac{1}{4}$ of an inch "goes into" $\frac{2}{3}$ of an inch. We get:

That is, just as in part (a) we could have divided 1" by 1/4", in part (b) we divide 2/3" by 1/4".

$$\frac{2}{3} \div \frac{1}{4} = \frac{2}{3} \times \frac{4}{1}$$

$$= \frac{8}{3}$$

$\frac{8}{3}$ $\begin{array}{r} 2 \text{ R2} \\ 3\overline{)8} \\ -6 \\ \hline 2 \end{array}$

We now take $\frac{8}{3}$ and multiply this by 75 feet to get:

$$\frac{8}{\underset{1}{\cancel{3}}} \times \overset{25}{\cancel{75}} \text{ feet} =$$

$$8 \times 25 \text{ feet} =$$

$$200 \text{ feet}$$

2 fourths of an inch represents 2 X 75 or 150 feet and 3 fourths of an inch represents 3 X 75 or 225 feet. Hence 2/3 of an inch represents more than 150 feet but less than 225 feet. Thus 200 feet is in the proper range.

Do the problem any way you like, but notice the importance of relative size again. To do this problem we had to compare the size of $\frac{2}{3}$ inches with $\frac{1}{4}$ inch. Since every $\frac{1}{4}$ inch represented 75 feet, we had to see how many *times* $\frac{1}{4}$ was contained within $\frac{2}{3}$. This involves division, *not subtraction.* The only difference between parts (a) and (b) is that in (a) the quotient was a whole number while in (b) it wasn't. Yet the technique is mathematically the same in both parts.

This is the key point. Whenever you want to see how many times larger one number is than another, you have to divide.

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Step 7:

Do Self-Test 5, Form B on the next page.

Self-Test 5, Form B

ANSWERS:

1. Which of the following quotients names the greatest rational number:

   (a) 3 ÷ 4 (b) 17 ÷ 24 (c) 5 ÷ 6?

   1. ____________

2. Which of the following represents the cheapest price per pound:
   (a) 40 pounds for $25.20?
   (b) 30 pounds for $19.50?
   (c) 50 pounds for $30.50?

   2. ____________

3. (a) How much is $\frac{4}{13} \times \frac{5}{9}$?

   (b) What must you multiply $\frac{4}{13}$ by to get $\frac{5}{9}$?

   3. (a) ____________

   (b) ____________

4. Which is more and by how much:

   2/5 of 3/7 or 3/10 of 5/14?

   4. ____________

5. A recipe calls for 7/8 of a cup of sugar. You only want to make 2/3 of the recipe. How much sugar should you use?

   5. ____________

6. Your friend buys 2/5 of a carton of books. You buy 4/7 of what's left. What fractional part of the carton did you buy?

   6. ____________

7. Write each of the following as a common fraction in lowest terms:

   (a) $\frac{3}{4} \times (\frac{2}{3} + \frac{4}{7})$

   (b) $(\frac{3}{4} \times \frac{2}{3}) + (\frac{3}{4} \times \frac{4}{7})$

   7. (a) ____________

   (b) ____________

8. Write each of the following as a common fraction in lowest terms:

   (a) $(\frac{8}{9} \div \frac{6}{7}) \div \frac{2}{7}$ (b) $\frac{8}{9} \div (\frac{6}{7} \div \frac{2}{7})$

   8. (a) ____________

   (b) ____________

9. An object travels 4/5 miles in 2/3 of a minute. What is the speed of the object in:

   (a) miles per minute? (b) miles per hour?

   9. (a) ____________

   (b) ____________

10. A map uses a scale of 1/8 of an inch to represent 50 feet. How many feet is represented by:

    (a) 1 inch? (b) 3/5 of an inch?

    10. (a) ____________

    (b) ____________

(ANSWERS ARE ON THE NEXT PAGE)

Answers For Self-Test 5, Form B

1. (c)

2. (c)

3. (a) $\frac{20}{117}$ (b) $\frac{65}{36}$

4. 2/5 of 3/7 by 9/140

5. 7/12 of a cup

6. 12/35

7. (a) $\frac{13}{14}$ (b) $\frac{13}{14}$

8. (a) $\frac{98}{27}$ (b) $\frac{8}{27}$

9. (a) $\frac{6}{5}$ miles per minute (b) 72 miles per hour

10. (a) 400 feet (b) 240 feet

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If you did each problem in Form B correctly, you may, if you wish, proceed to the next module. Otherwise, continue with Step 8.

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Step 8:

View the solutions for Self-Test 5, Form B on Videotape Lecture 5S. Pay special attention to the solutions of those problems for which you failed to get the correct answers. Feel free to rewind the tape at any time to restudy the problems that gave you difficulty.

Step 9:

Do Self-Test 5, Form C on the next page.

Self-Test 5, Form C

ANSWERS:

1. Which of the following quotients names the greatest rational number:

   (a) 5 ÷ 8 (b) 11 ÷ 20 (c) 7 ÷ 12?

   1. ____________

2. Which of the following represents the cheapest price per pound:

   (a) 40 pounds for $24.40?
   (b) 70 pounds for $35.70?
   (c) 60 pounds for $29.40?

   2. ____________

3. (a) How much is $\frac{5}{11} \times \frac{4}{9}$?

   (b) What must you multiply $\frac{5}{11}$ by to get $\frac{4}{9}$?

   3. (a) ____________

   (b) ____________

4. Which is more and by how much:

   3/4 of 5/6 or 7/8 of 7/12?

   4. ____________

5. A recipe calls for 15/16 of a cup of flour. You only want to make 2/3 of the recipe. How much flour should you use?

   5. ____________

6. Your friend buys 2/9 of a carton of books. You buy 1/3 of what's left. What fractional part of the carton did you buy?

   6. ____________

7. Write each of the following as a common fraction in lowest terms:

   (a) $\frac{2}{5} \times (\frac{3}{4} + \frac{4}{9})$ (b) $(\frac{2}{5} \times \frac{3}{4}) + (\frac{2}{5} \times \frac{4}{9})$

   7. (a) ____________

   (b) ____________

8. Write each of the following as a common fraction in lowest terms:

   (a) $(\frac{3}{5} \div \frac{4}{9}) \div \frac{2}{9}$ (b) $\frac{3}{5} \div (\frac{4}{9} \div \frac{2}{9})$

   8. (a) ____________

   (b) ____________

9. An object travels 2/3 of a mile in 5/7 of a minute. What is the speed of the object in:

   (a) Miles per minute? (b) miles per hour?

   9. (a) ____________

   (b) ____________

10. A map uses a scale of 1/3 of an inch to represent 70 feet. How many feet is represented by:

    (a) 1 inch? (b) $\frac{2}{7}$ of an inch?

    10. (a) ____________

    (b) ____________

(ANSWERS ARE ON THE NEXT PAGE)

Answers for Self-Test 5, Form C

1. (a)
2. (c)
3. (a) $\frac{20}{99}$ (b) $\frac{44}{45}$
4. 3/4 of 5/6 by 11/96
5. 5/8 of a cup
6. 7/27
7. (a) 43/90 (b) 43/90
8. (a) 243/40 (b) 3/10
9. (a) 14/15 miles per minute (b) 56 miles per hour
10. (a) 210 (b) 60

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THIS CONCLUDES OUR STUDY GUIDE PRESENTATION FOR MODULE #5.

HOPEFULLY, YOU WILL NOW FEEL READY TO BEGIN MODULE #6.

HOWEVER, IF YOU STILL FEEL UNCERTAIN OF THE MATERIAL IN THIS MODULE, YOU SHOULD CONSULT WITH A TEACHER, A FRIEND, OR A FELLOW-STUDENT FOR ADDITIONAL REINFORCEMENT.

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